LISA as a Xylophone Interferometer Detector of Gravitational Radiation

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Abstract. A filtering technique, for reducing the frequency fluctuations of the laser entering into the two-way Doppler tracking data measured with two spacecraft, is discussed. This method takes advantage of the sinusoidal behavior of the transfer function of this noise source to the Doppler observable, which displays sharp nulls at selected Fourier components. The non-zero gravitational wave signal remaining at these frequencies makes spacecraft to spacecraft laser Doppler tracking the equivalent of a *xylophone interferometer* detector of gravitational radiation.

The data analysis technique presented in this paper could be implemented with the LISA mission at the Fourier frequencies where the algorithm for unequal-arm interferometers fails to work, or as a backup option in case of failure of one of the three spacecraft.

INTRODUCTION

Non-resonant detectors of gravitational radiation (with frequency content $0 < f < f_H$) are essentially interferometers, in which a coherent train of electromagnetic waves (of nominal frequency $\nu_0 \gg f_H$) is folded into several beams, and at points where these intersect relative fluctuations of frequency or phase are monitored (homodyne detection). The observed low frequency signal is due to frequency variations of the source of the beams about ν_0, to relative motions of the source and the mirrors (or amplifying transponders) that do the folding, to temporal variations of the index of refraction along the beams, and, according to general relativity, to any time-variable gravitational fields present, such as the transverse traceless metric curvature of a passing plane gravitational wave train. To observe these gravitational fields in this way, it is thus necessary to control, or monitor, the other sources of relative frequency fluctuations, and, in the data analysis, to optimally use algorithms based on the different characteristic interferometer responses to gravitational waves (the signal) and to the other sources (the noise) [1].

Space-based interferometers, such as the coherent microwave tracking of interplanetary spacecraft [2] and proposed Michelson optical interferometers in planetary orbits [3], are most sensitive to milliHertz gravitational waves and have arm lengths ranging from 10^6 to 10^8 kilometers.

In present single-spacecraft Doppler tracking observations many of the noise sources can be either reduced or calibrated by implementing appropriate microwave frequency links and by using specialized electronics, so the fundamental limitation is imposed by the frequency (time-keeping) fluctuations inherent to the reference clocks that control the microwave system. Hydrogen maser clocks, currently used in Doppler tracking experiments, achieve their best performance at about 1000 seconds integration time, with a fractional frequency stability of a few parts in 10^{-16}. This is the reason why these one-arm interferometers in space are most sensitive to milliHertz gravitational waves. This integration time is also comparable to the microwave propagation (or "storage") time $2L/c$ to spacecraft en route to the outer solar system ($L \simeq 3AU$) [4].

By comparing phases of split beams propagated along non-parallel arms, source frequency fluctuations can be removed and gravitational wave signals at levels many orders of magnitude lower can be detected. Especially for space-based interferometers such as LISA, that use lasers with a frequency stability of a few parts in 10^{-13}, it is essential to be able to remove these fluctuations when searching for gravitational waves of dimensionless amplitudes less than 10^{-20} in the milliHertz band.

Since the armlengths of these space-based interferometers can be different by several percent, the direct recombination of the two beams at a photo detector will not remove completely the laser noise. This is because the frequency fluctuations of the laser will be delayed by a different amount of time into the two different-length arms. In order to solve for this problem, a technique involving heterodyne interferometry with unequal arm lengths and independent readouts was proposed [5], which yielded data from which source frequency fluctuations were removed by many orders of magnitude. The technique discussed in [5], however, is not effective at frequencies equal to integer multiples of the inverse of the round trip light times in the two arms. This is because the transfer functions of the laser fluctuations to the Doppler tracking responses have sharp nulls at these frequencies. This implies that the information about the laser fluctuations provided by one of the two Doppler responses at these Fourier frequencies is lost, and the calibration of these fluctuations at these frequencies can not be performed [5].

In this paper we will show, however, that it is still possible to make measurements of gravitational radiation at these frequencies by using only the data generated by a single pair of spacecraft. In this sense we can regard spacecraft to spacecraft coherent laser tracking as a *xylophone interferometer* detector of gravitational radiation [6]. An outline of the paper is presented below.

In section II, after deriving the transfer functions of the noise sources affecting the two-way tracking data set, we show that there exist selected Fourier frequencies at which the transfer function of the laser frequency fluctuations into the Doppler observable is essentially null. These are what we will refer to as the frequencies of the xylophone.

Since the xylophone frequencies are equal to integer multiples of the inverse of the round trip light time, any variation in the distance between the spacecraft implies changes in these frequencies. For the selected LISA trajectory, we can successfully implement the xylophone technique by integrating the data for time intervals during which the variations of the xylophone frequencies are smaller than the frequency resolution bin. Estimates of the maximum integration times allowed by each of the three pairs of spacecraft are presented.

The noise levels, achievable with this technique with two LISA spacecraft, are presented in section III. We find that a strain sensitivity of 2.5×10^{-21} at the frequency 3×10^{-2} Hz can be reached when searching for sinusoids and by integrating the data for 10 days. At this sensitivity level, gravitational radiation from galactic binary systems should be observable. Our comments and conclusions are then presented in section IV.

SPACECRAFT TO SPACECRAFT COHERENT LASER TRACKING AS A XYLOPHONE INTERFEROMETER

Let us consider two of the three LISA spacecraft, each acting as a free falling test particle, and continuously tracking each other via coherent laser light. One spacecraft, which we will refer to as spacecraft a, transmits a laser beam of nominal frequency ν_0 to the other spacecraft (spacecraft b). The phase of the light received at spacecraft b is used by a laser on board spacecraft b for coherent transmission back to spacecraft a. The relative two-way frequency (or phase) changes $\Delta\nu/\nu_0$, as functions of time, are then measured at a photo detector. When a gravitational wave crossing the solar system propagates through this electromagnetic link, it causes small perturbations in $\Delta\nu/\nu_0$, which are replicated three times in the Doppler data with maximum spacing given by the two-way light propagation time between the two spacecraft.

Let us introduce a set of Cartesian orthogonal coordinates (X, Y, Z) centered on one of the two spacecraft, say spacecraft a. The Z axis is assumed to be oriented along the direction of propagation of a gravitational wave pulse, and (X, Y) are two orthogonal axes in the plane of the wave (see Figure 1). In this coordinate system we can write the two-way Doppler response, measured by spacecraft a at time t, as follows

$$\begin{aligned}\left(\frac{\Delta\nu(t)}{\nu_0}\right)_a \equiv y(t) = & -\frac{(1-\mu)}{2}\, h(t) \;-\; \mu\, h(t-(1+\mu)L) + \frac{(1+\mu)}{2}\, h(t-2L) \\ & + C_a(t-2L) \;-\; C_a(t) \;+\; 2B_b(t-L) \;+\; B_a(t-2L) \;+\; B_a(t) \\ & + TR_b(t-L) \;+\; N_{2a}(t)\;, \qquad (1)\end{aligned}$$

where $h(t)$ is equal to

$$h(t) = h_+(t)\cos(2\phi) + h_\times(t)\sin(2\phi) \ . \tag{2}$$

Here $h_+(t)$, $h_\times(t)$ are the wave's two amplitudes with respect to the (X, Y) axis, (θ, ϕ) are the polar angles describing the location of spacecraft b with respect to the (X, Y, Z) coordinates, μ is equal to $\cos\theta$, and L is the distance between the two spacecraft (units in which the speed of light $c = 1$).

We have denoted with $C_a(t)$ the random process associated with the frequency fluctuations of the laser on board spacecraft a; $B_a(t)$, $B_b(t)$ are the joint effects of the noises from buffeting by non gravitational forces on the test masses on board spacecraft a and b respectively, $TR_b(t)$ is the noise due to the optical transponder on board spacecraft b, and $N_{2a}(t)$ is the noise from the photo detector on board spacecraft a where two-way phase changes are measured.

From Eq. (1) we deduce that gravitational wave pulses of duration longer than the round trip light time $2L$ have a Doppler response $y(t)$ that, to first order, tends to zero. The tracking system essentially acts as a pass-band device, in which the low-frequency limit f_l is roughly equal to $(2L)^{-1}$ Hz, and the high-frequency limit f_H is set by the shot noise at the photo detector [2,4].

In Eq. (1) it is also important to note the characteristic time signatures of the random processes $C_a(t)$, $B_a(t)$, and $B_b(t)$. The time signature of the noise $C_a(t)$ can be understood by observing that the frequency of the signal received at time t

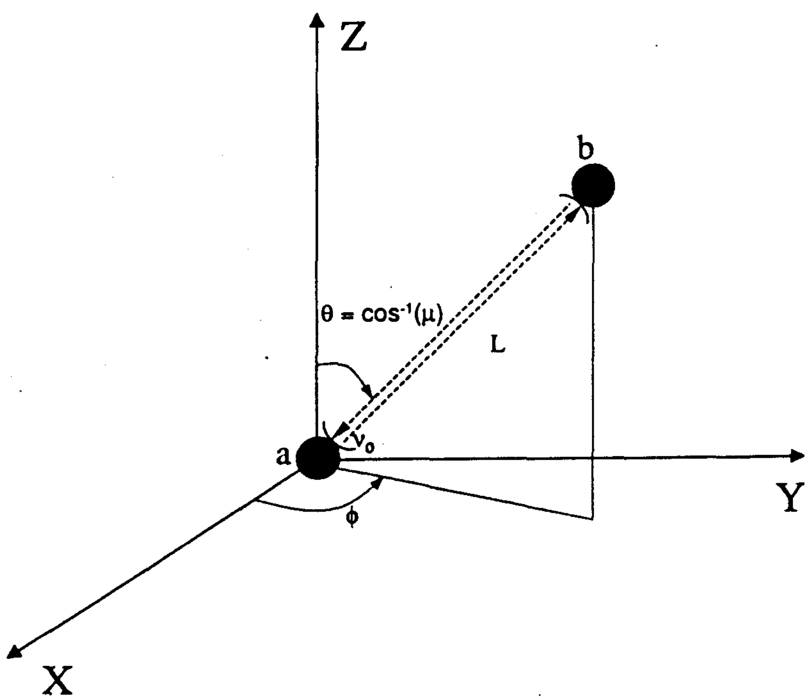

FIGURE 1. Coherent laser light of nominal frequency ν_0 is transmitted from spacecraft a to spacecraft b, and coherently transponded back. The gravitational wave train propagates along the Z direction, and the cosine of the angle between its direction of propagation and the laser beam is denoted by μ.

contains laser frequency fluctuations transmitted $2L$ seconds earlier. By subtracting from the frequency of the received signal the frequency of the signal transmitted at time t, we also subtract the frequency fluctuations $C_a(t)$ with the net result shown in Eq. (1). As far as the fluctuations due to buffeting of the test-mass on board spacecraft a are concerned, the frequency of the received signal is affected at the moment of reception as well as $2L$ seconds earlier. Since the frequency of the signal generated at time t does not contain yet any of these fluctuations, we conclude that $B_a(t)$ is positive-correlated at the round trip light time $2L$. The time signature of the noise $B_b(t)$ in Eq. (1) can be understood through similar considerations [6].

Among all the noise sources included in Eq. (1), the frequency fluctuations due to the lasers are expected to be the largest. A space-qualified single-mode laser, such as a diode-pumped Nd:YAG ring laser of frequency $\nu_0 = 3.0 \times 10^{14}$ Hz and phase-locked to a Fabry-Perot optical cavity, is expected to have a spectral level of frequency fluctuations equal to about $1.0 \times 10^{-13}/\sqrt{Hz}$ in the milliHertz band. Frequency stability measurements performed on such a laser by McNamara *et al.* [7] indicate that a stability of about $1.0 \times 10^{-14}/\sqrt{Hz}$ might be achievable in the same frequency band.

For the moment we will not make any assumptions on the frequency stability of the onboard laser, and return to this point later. We will focus instead on its transfer function and on the transfer function of the gravitational wave signal as shown in Eq. (1). If we denote with $\widetilde{y}(f)$ the Fourier transform of the time series $y(t)$, defined as

$$\widetilde{y}(f) \equiv \int_{-\infty}^{+\infty} y(t)\ e^{2\pi i f t}\ dt\ , \tag{3}$$

then Eq. (1) can be rewritten in the Fourier domain as follows

$$\begin{aligned} \widetilde{y}(f) = & \left[\frac{(\mu - 1)}{2} - \mu\ e^{2\pi i f(1+\mu)L} + \frac{(1+\mu)}{2}\ e^{4\pi i f L}\right] \widetilde{h}(f) \\ & + \widetilde{C_a}(f)\ \left[e^{4\pi i f L} - 1\right] + \widetilde{B_a}(f)\ \left[e^{4\pi i f L} + 1\right] + 2\ \widetilde{B_b}(f)\ e^{2\pi i f L} \\ & + \widetilde{TR_b}(f)\ e^{2\pi i f L} + \widetilde{N_a}(f)\ . \end{aligned} \tag{4}$$

Note that the transfer function of the noise C_a is equal to zero at frequencies that are integer multiples of the inverse of the round trip light time, while the transfer function of the gravitational wave signal is in general different from zero. By making coherent laser tracking measurements at these frequencies, we are in fact making *xylophone interferometric measurements* of gravitational waves.

If we define Δf to be the frequency resolution of our data set (equal to the inverse of the integration time τ), to first order in $(\Delta f\ L)$ and at the xylophone frequencies $f_k = k/2L$, the response $\widetilde{y}(f_k)$ can be approximated by the following expression [6]

$$\begin{aligned} \widetilde{y}(f_k) \simeq & \ \mu\ \left[1 + (-1)^{k+1} e^{\pi i k \mu}\right]\ \widetilde{h}(f_k)\ \pm\ \widetilde{C_a}(f_k)\ (2\Delta f L) \\ & + 2\left[\widetilde{B_a}(f_+) + \widetilde{B_b}(f_+)\right] + (-1)^k\ \widetilde{TR_b}(f) + \widetilde{N_a}(f_k)\ , \end{aligned} \tag{5}$$

If we take $L = 5 \times 10^6$ km, and assume an integration time τ of three months as a numerical example, we find that the amplitudes of the frequency fluctuations due to the laser are reduced at the xylophone frequencies by a factor of

$$\frac{2\,\Delta f\ L}{c} = 4.1 \times 10^{-6}\ . \tag{6}$$

Eq. (5) shows some interesting, and somewhat peculiar, properties of the remaining gravitational wave signal at the xylophone frequencies. The response to a gravitational wave pulse goes to zero not only when the wave propagates along the line of sight between the spacecraft ($\mu = \pm 1$), but also for directions orthogonal to it ($\mu = 0$). This is consequence of the fact that for $\mu = 0$ the Doppler response y to a gravitational wave becomes a *two-pulse* response, identical to the response of the laser noise, and therefore it cancels out at the xylophone frequencies.

For sources randomly distributed in the sky, as in the case of a stochastic background of gravitational waves, we can assume the angles (θ, ϕ) to be random variables uniformly distributed over the sphere. Since the average over (θ, ϕ) of the response given in Eq. (4) is equal to zero, it follows that the variance (denoted with $\Sigma^2(f)$) of the antenna pattern is equal to [2]

$$\Sigma^2(f) = \frac{(2\pi f L)^2 - 3}{(2\pi f L)^2} - \frac{(2\pi f L)^2 + 3}{3(2\pi f L)^2}\ \cos(4\pi f L) + \frac{2}{(2\pi f L)^3}\ \sin(4\pi f L)\ . \tag{7}$$

At the xylophone frequencies f_k, Σ^2 becomes the following monotonically increasing function of the integer k

$$\Sigma^2(k) = \frac{2}{3} - \frac{4}{(\pi k)^2}\ , \tag{8}$$

ESTIMATED SENSITIVITIES

In order to take advantage of the xylophone technique it is necessary to integrate over a sufficiently long period of time. This is because we want to reduce the noise due to the laser to a level as close as possible to that identified by the remaining noise sources at the xylophone frequencies (see Eq. (5)). Since the xylophone frequencies change in time as the distance between the spacecraft varies, we can not coherently integrate our data indefinitely. Coherent integration can be performed only on a time scale τ during which the variations of the xylophone frequencies are smaller than the frequency resolution $\Delta f = 1/\tau$.

In order to identify the maximum time of coherent integration for our xylophone interferometer detector, we need to identify the time dependence of the separation $L(t)$ between two of the LISA spacecraft. From the definition of the frequencies f_k, we can then derive the variation of the xylophone frequencies, δf_k, in terms of the relative change in the distance between the spacecraft, $\delta L(t)/L(0)$. Since δf_k is related to $\delta L(t)$ by the following equation [6]

$$\delta f_k = \frac{k}{2L} \times \frac{\delta L(t)}{L(0)} \,, \tag{9}$$

it follows that the maximum time of integration can be computed by requiring δf_k to be smaller than the frequency resolution $\Delta f = 1/\tau$.

It has been calculated by Folkner *et al.* [8] that the relative longitudinal speeds between the three pairs of spacecraft, during approximately the first year of the mission, can be written in the following approximated form

$$V_{i,j}(t) = V_{i,j}^{(0)} \sin\left(\frac{2\pi t}{T_{i,j}}\right) \qquad (i,j) = (a,b) \ ; \ (a,c) \ ; \ (b,c) \,, \tag{10}$$

where we have denoted with $(a,b), (a,c), (b,c)$ the three possible spacecraft pairs, $V_{i,j}^{(0)}$ is a constant velocity, and $T_{i,j}$ is the period for the pair (i,j). In reference [8] it has also been shown that the LISA trajectory can be selected in such a way that two of the three arms' rates of change are essentially equal during the first year of the mission. This configuration is particularly attractive because it implies an almost null variation in differential armlength for one of the three interferometers. Following reference [8], we will assume $V_{a,b}^{(0)} = V_{a,c}^{(0)} \neq V_{b,c}^{(0)}$, with $V_{a,b}^{(0)} = 1$ m/s, $V_{b,c}^{(0)} = 13$ m/s, $T_{a,b} = T_{a,c} \approx 4$ months, and $T_{b,c} \approx 1$ year. With these numerical values we can calculate the maximum integration times for different xylophone frequencies. The calculation is straightforward, and we will not go through it here [6]. The results of this analysis, however, indicate that the data from the two pairs of spacecraft, $(a,b), (a,c)$, can be integrated coherently at the frequency $f_1 = 3 \times 10^{-2}$ Hz for about 10 days. A shorter integration time of about 3 days is needed instead to make xylophone measurements at the frequency 1.5 Hz. For the remaining pair of spacecraft, due to their larger relative speed, we have found that coherent integration at f_1 can be performed for about 6 days, while at 1.5 Hz the maximum integration time goes down to about 2 days.

The numerical values of the maximum integration times derived above allow us to estimate, at the xylophone frequencies, the one-sided power spectral density of the noise affecting the data set y. In what follows we will consider two spacecraft with identical optical and mechanical payloads, and equal to those described in [3]. We will also assume the random processes associated with the noise sources affecting the stability of the coherent one-way tracking data to be uncorrelated with each other, and their one-sided power spectral densities to be consistent with those given in reference [3]. Since our xylophone will be sensitive to gravitational radiation at frequencies equal to or larger than the inverse of the round trip light time ($c/2L \approx 3 \times 10^{-2}$ Hz), the dominant noise sources determining its strain sensitivity will be the photon-shot noise, and the frequency fluctuations of the laser [3].

After taking into account Eq. (5), and the expressions of the one-sided power spectral density for the shot-noise and the frequency fluctuations of the laser given in reference [3], the one-sided power spectral density $S_y(f)$ of the noise in the response y, estimated in the frequency band of the xylophone, can be written as follows

$$S_y(f) = 10^{-38}\, f^2 + \left[10^{-28}\, f^{-2/3} + 6.3 \times 10^{-37}\, f^{-3.4}\right] \sin^2(2\pi f L)\ , \qquad (11)$$

In Figure 2 we have plotted this function by assuming an integration time of 3 days. Note that, with such an integration time, the one-sided power spectral density of the laser noise is reduced, at the xylophone frequencies, by a factor of $(2\Delta f\ L)^2 = 1.6 \times 10^{-8}$. Since the function $S_y(f)$ plotted in Figure 2 is monotonically decreasing at the xylophone frequencies, we conclude that with such an integration time the noise due to the laser is still the dominant one.

The 3 days time interval implies a variation of the largest of the xylophone frequencies smaller than the frequency resolution bin. At smaller xylophone frequencies, however, the one-sided power spectral densities should be rescaled according to the appropriate maximum integration times. For instance, since at the frequency 3×10^{-2} Hz we can coherently integrate the data from two of the three pairs of spacecraft for 10 days, the one-sided power spectral density at this frequency would be smaller than the value shown in Figure 2 by a factor $(3/10)^2$. From the estimate

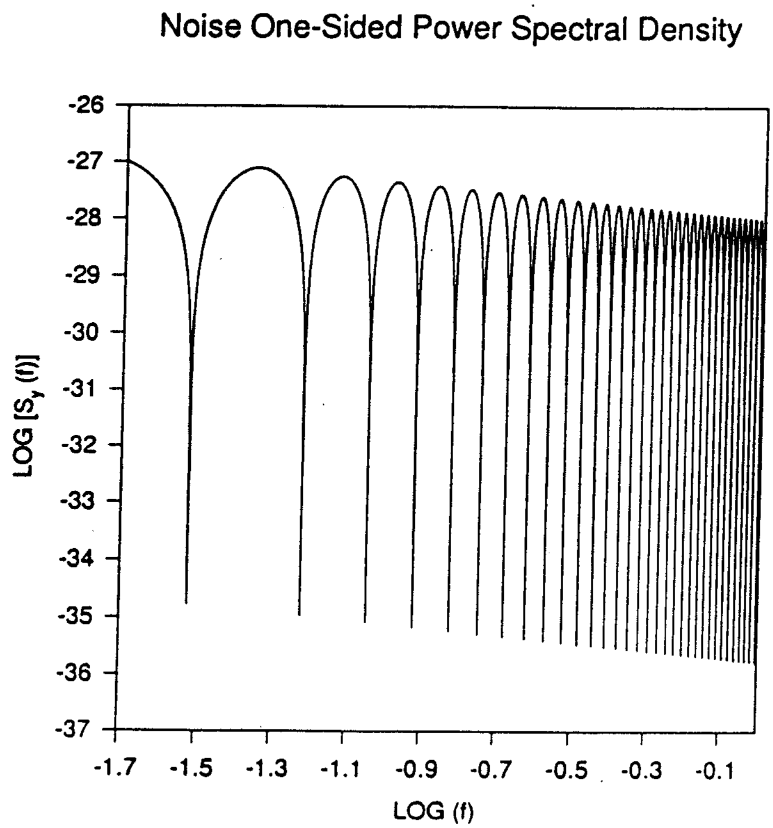

FIGURE 2. The one-sided power spectral density of the noise, $S_y(f)$, entering into the response y. The minima of S_y have been estimated by assuming an integration time of 3 days. See text for explanation.

of the one-sided power spectral density of the noise given in Figure 2, it is possible to calculate the root-mean-squared (r.m.s.) noise level $\sigma(f_k)$ of the frequency fluctuations in the bins of width Δf, around the frequencies f_k ($k = 1, 2, 3, ...$). This is given by the following expression

$$\sigma(f_k) \equiv [S_y(f_k)\ \Delta f]^{1/2} \ , \ k = 1, 2, 3, ... \ . \tag{12}$$

This measure of the Doppler sensitivity is appropriate for sinusoidal signals, such as those generated during the coalescence of a binary system, while it over estimates the sensitivity to bursts and to a stochastic background of gravitational radiation. The quantitative results implied by the formula given in Eq. (12) should therefore be considered by keeping in mind this observation. For a detailed and quantitative analysis covering bursts and stochastic waveforms instead, the reader is referred to [6].

From Eqs. (5, 8) we derive that, at 3×10^{-2} Hz, the xylophone will have an r.m.s. strain sensitivity to sinusoids, averaged over an isotropic distribution of source directions, of about 2.6×10^{-21}. A binary system containing two 1.0 $M_{\odot}$ stars, for instance, could radiate sinusoidally at $f = 3 \times 10^{-2}$ Hz, and during a period of 17 days the frequency of the radiation would change by an amount smaller than the frequency resolution of the data. Since the Doppler data can be integrated coherently for about 10 days at the xylophone's fundamental, we find that such a binary system could be observed out to a distance of about 3 kpc.

CONCLUSIONS

We have discussed a data analysis technique for performing searches of gravitational radiation in space with two spacecraft tracking each other via coherent laser light. The main result of our analysis, deduced in Eq. (5), shows that we can reduce, by several orders of magnitude, the frequency fluctuations introduced in the Doppler data by the laser. This is achieved by making measurements at the Fourier frequencies where the transfer function of the laser fluctuations to the Doppler observable has sharp minima. In this respect spacecraft to spacecraft coherent laser tracking can be regarded as a *xylophone interferometer* detector of gravitational radiation.

When searching for sinusoids, we have found that a strain sensitivity of 2.6×10^{-21} at the frequency $3 \ \times 10^{-2}$ Hz can be obtained after coherently integrating the two-way Doppler data for 10 days. At this sensitivity level, gravitational radiation from galactic coalescing binary systems should be observable.

Spacecraft to spacecraft xylophone interferometric measurements of gravitational radiation could be implemented with the LISA mission at the Fourier frequencies where the algorithm for unequal-arm interferometers fails to work, or as backup option in case of failure of one of the three spacecraft.

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